Generalized Attributes in Concept Lattices

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joint work with R. Kuitché and E. R. Temgoua
UYI and ENS, Yaoundé - Cameroon

Elementary Information system: Contexts

$\mathbb{K}$	a	b	c	d	e	f	g	h
1	×				×		×	
2	×				×	×		×
3	×	×			×	×	×	
4		×			×	×	×	×
5	×		×	×				
6	×	×	×	×				
7		×	×				×	
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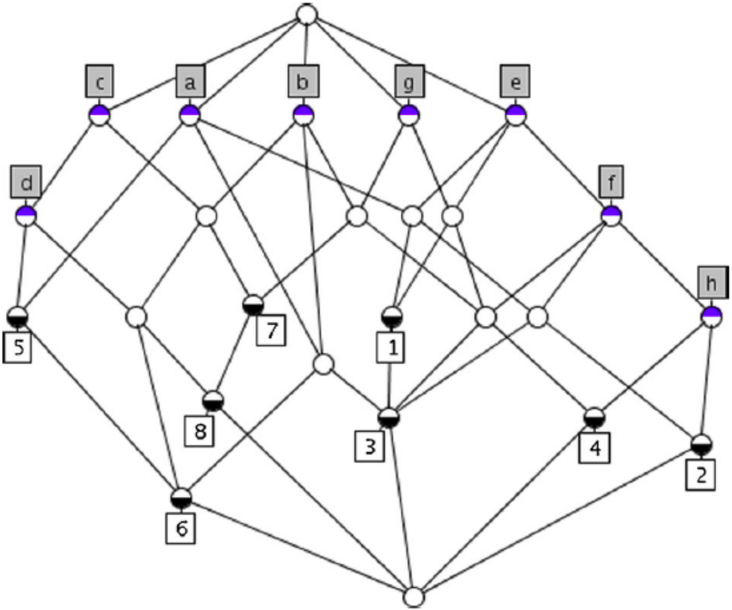
Lattice of concepts



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- A **formal concept** of $\mathbb{K}$ is a pair (A, B) with $A' = B$ and $B' = A$.
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- $c : X \mapsto X''$ is a closure operator on $\mathcal{P}(G)$ and on $\mathcal{P}(M)$.
- $\mathrm{Ext}(\mathbb{K}) := c(\mathcal{P}(G)) \cong^d c(\mathcal{P}(M)) =: \mathrm{Int}(\mathbb{K})$.
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- In data mining, generalized patterns are pieces of knowledge extracted from data when an ontology is used. For example the attributes of $\mathbb{K}$ can be grouped together to form set S of new attributes.

- In basket market analysis, items or products can be grouped into product lines or product categories. Customers may be grouped according to some specific features (e.g., income, education).

- By grouping the attributes of $\mathbb{K}$, we actually replace (G, M, I) with a new context (G, S, J) with S covering M and J to be precised.

- There are mainly three ways to express the relation J:
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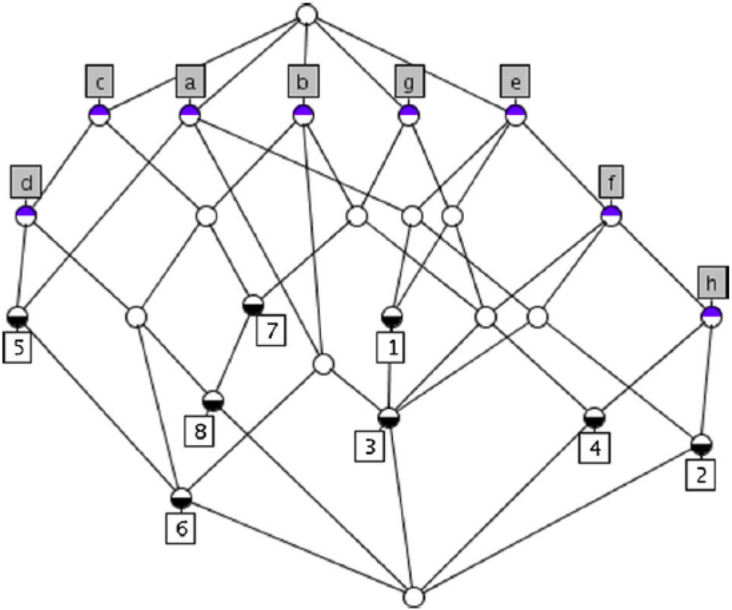
Lattice of concepts



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The generalized attributes are

(∃) $A := \{e, g\}$, $B := \{b, c\}$, $C := \{a, d\}$, $D := \{f, h\}$.

(∀) $S := \{e, g\}$, $T := \{b, c\}$, $U := \{a, d\}$, $V := \{f, h\}$.

(α) $E := \{a, b, c\}$, $F := \{d, e, f\}$, $H := \{g, h\}$ with threshold $\alpha = 60\%$

Expected Gain

Generalizing attributes reduced the size of the context. So we expect also the size of the concept lattice to reduce. BUT this is not always the case.

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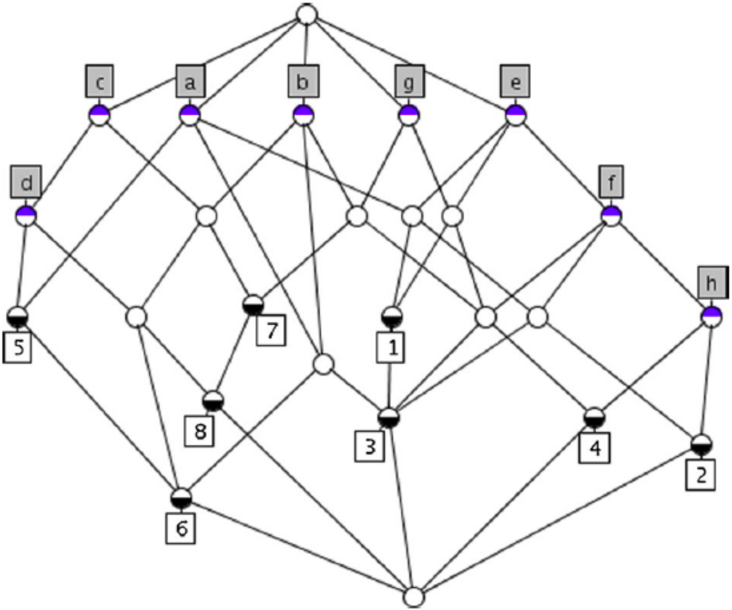
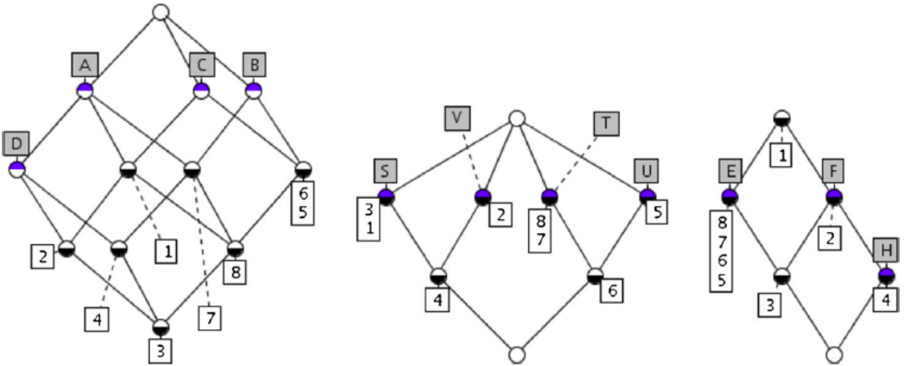
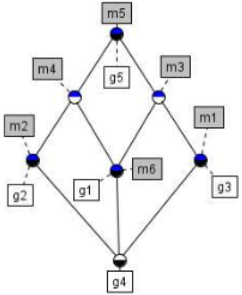


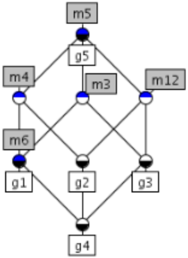
Lattice of concepts with generalized attributes

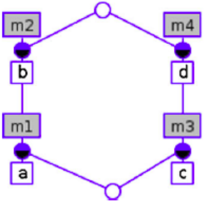


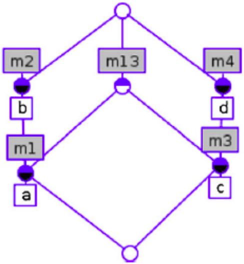
The size decreases in both three cases.

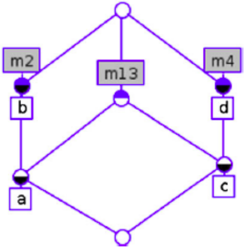
Generalizing attributes: the size can increase!











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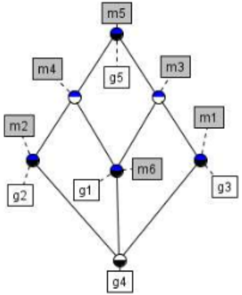

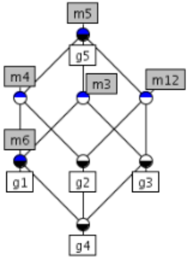

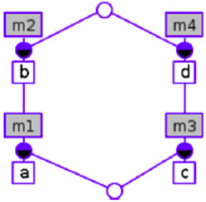

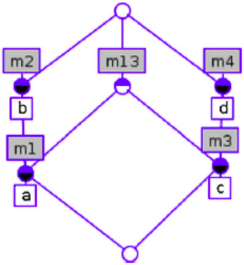

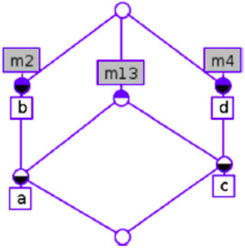

Observation and questions

- The ∀-generalizations on attributes do not increase the size of the concept lattice.
- If the concept lattice is distributive, then any ∃-generalization reduces the size of the initial lattice.
- The lattice B_4 is the smallest lattice on which there is an ∃-generalization that increases the size of the initial concept lattice.

Questions

- Can the size increase by more than one after a ∃-generalisation?
- Can the size remains unchanged after a ∃-generalisation?
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The size can increase exponentially!

- For any set S, the context $(S, S, \neq)$ has $2^{|S|}$ concepts, that form a Boolean algebra.
- Set $S_n = \{1, \ldots, n\}$. Let $g_1, m_1, m_2 \notin S_n$.
- Set $\mathbb{K}_n^k := (S_n \dot\cup \{g_1\}, S_n \dot\cup \{m_1, m_2\}, \mathrm{I})$ with $k \in S_n$ and
  - $\mathrm{I} \cap (S_n \times S_n) = \neq$
  - $g_1' = S_n, \quad m_1' = \{1, \ldots, k\}$ and $m_2' = S_n \setminus m_1'$.

- The context resulting from a $\exists$-generalization of m_1 and m_2 is isomorphic to $(S_{n+1}, S_{n+1}, \neq)$ and therefore has 2^{n+1} concepts.
- The context $\mathbb{K}_n^k$ has $2^n + 2^k + 2^{n-k} - 1$ concepts.
- Putting m_1 and m_2 together increases the size by $2^n - 2^k - 2^{n-k} + 1$.
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Is this the worst case?

$$g(k) = 2^n - 2^k - 2^{n-k} + 1$$
$$0 = g'(k) = -\ln(2)2^k + \ln(2)2^{n-k} \iff n = 2k.$$
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- For any context (G, M, I), the number of concept is $\leq 2^{\min(|M|,|G|)}$.
- Let $(G, M \cup \{a, b\}, I)$ be a context and $(G, M \cup \{ab\}, I)$ the context obtained by $\exists$-generalizing a and b.
- $\begin{cases} |\mathfrak{B}(G, M, I)| \leq |\mathfrak{B}(G, M \cup \{a, b\}, I)| \\ |\mathfrak{B}(G, M, I)| \leq |\mathfrak{B}(G, M \cup \{ab\}, I)| \end{cases}$
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  is an injective map.
- If $a' = G$ then Φ_a is a bijection.
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