92. Arbeitstagung Allgemeine Algebra

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ABSTRACTS

Invited talks

Covering a group by conjugates of a coset

Barbara Baumeister
Bielefeld University

For every doubly transitive permutation group G the conjugates of a non-trivial coset of a point stabilizer H cover the group. This implies that every non-trivial conjugacy class of elements of G that contains an element of H does contain a transversal of H in G and that every other non-trivial conjugacy class contains a transversal for the set of cosets of H in G different from H.

We study the finite groups satisfying this property; more precisely the class of primitive permutation groups, called CCI groups, that in fact properly contains the class of 2-transitive permutation groups.

Multifraction reduction in Artin-Tits groups

Patrick Dehornoy
University of Caen

A classical result of Ore says that, if M is a cancellative monoid and any two elements of M admit a least common multiple, then every element of the enveloping group $U(M)$ of M can be represented by a unique irreducible fraction on M. We extend this result by showing that, when common multiples need not exist but a certain "3-Ore condition" is satisfied, every elements of $U(G)$ can be represented by a unique irreducible iterated fraction, leading to a solution of the Word Problem reminiscent of the Dehn algorithm for hyperbolic groups. This

applies in particular to Artin-Tits groups of FC-type and, conjecturally, to all Artin-Tits groups.

Representing groups by endomorphisms of the random graph

Igor Dolinka
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Coauthors: R.D. Gray, J.D. McPhee, J.D. Mitchell and M. Quick

I will begin by introducing the so-called random graph R, a prime example of a countable ultrahomogeneous first-order structure. This will lead us to the outline of a beautiful subject in model theory, the Fraïssé theory, centred around the notion of an amalgamation class of structures.

The other basic ingredient of the talk will be semigroups, especially in the light of their links to groups via maximal subgroups of a semigroup and 'hidden' group structures called Schutzenberger groups (of a $\mathcal{D}$-class). Hence, I will provide a gentle crash-course (excuse the pun) into the basics of semigroup theory. These fundamentals will be then applied to endomorphism monoids of structures, naturally leading to the concept of an algebraically closed structure.

Our main results establish links between countable algebraically closed graphs and the endomorphisms of the random graph R. In particular, we show that, for any countable graph Γ there are uncountable many maximal subgroups of the endomorphism monoid of R isomorphic to the automorphism group of Γ. Further structural information about $\mathrm{End}(R)$ is established including that $\mathrm{Aut}(\Gamma)$ arises in uncountably many ways as a Schutzenberger group. Similar results hold for the countable universal directed graph and the countable universal bipartite graph.

Directed Jónsson and Gumm terms

Alexandr Kazda
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Coauthors: Marcin Kozik, Ralph McKenzie and Matt Moore

Directed Jónsson and Gumm terms are variants of the classic terms for congruence distributive and congruence modular varieties. Given an $n \in \mathbb{N}$, the chain of length n of directed Jónsson terms is:

$$\begin{aligned} &d_1(x,x,y) \approx x, \\ &d_i(x,y,x) \approx x && \text{for } 1 \leq i \leq n, \\ &d_i(x,y,y) \approx d_{i+1}(x,x,y) && \text{for } 1 \leq i < n, \\ &d_n(x,y,y) \approx y, \end{aligned}$$

while the chain of $n+1$ directed Gumm terms is:

$$\begin{aligned} d_1(x,x,y) &\approx x, \\ d_i(x,y,x) &\approx x && \text{for } 1 \leq i \leq n, \\ d_i(x,y,y) &\approx d_{i+1}(x,x,y) && \text{for } 1 \leq i < n, \\ d_n(x,y,y) &\approx p(x,y,y) \\ p(x,x,y) &\approx y. \end{aligned}$$

Besides being aestheticaly pleasant, these directed terms are often more comfortable to use than the classic ones; directed Gumm terms appear eg. in Libor Barto's proof of the Valeriote conjecture.

It is straightforward to show that if a variety V admits directed Jónsson resp. Gumm terms then V is congruence distributive resp. modular. We will explain how to go about proving the converse implication.

Qualitative Calculi as a generalisation of Tarski's Relation Algebras

Tomasz Kowalski
La Trobe University, Melbourne
Coauthors: Robin Hirsch and Marcel Jackson

Qualitative Calculi arose in certain areas of Artificial Intelligence, as a means of modelling the natural (human, sloppy) reasoning, mostly about time and space. Accordingly, the two most famous Qualitative Calculi are: Allen's Interval Algebra (for reasoning about time intervals), and Region Connection Calculus (for reasoning about space). From the very beggining it was realised that Qualitative Calculi bear a connection to Tarski's Relation Algebras, but the mathematical foundations of this observation were not explored. We will explore them in some detail, touching upon some universal algebra, some model theory and some computational complexity.

Contributed talks

Congruence lattices forcing nilpotency

Erhard Aichinger
Johannes Kepler University Linz

We start from the following results, which are consequences of theorems by Freese, Hobby, and McKenzie.

> Let $\mathbf{A}$ be a finite algebra in a congruence modular variety such that $\mathrm{Con}(\mathbf{A})$ has a $(0,1)$-sublattice $\mathbb{L}$ that is simple, complemented, and has at least three elements. Then $\mathbf{A}$ is abelian.

Similarly if $\mathbb{L}$ has no 2-element homomorphic image, then $\mathbb{A}$ is solvable. We derive a similar condition for nilpotency and investigate what could be converses to these results.

On intertwining non normal operators in a Banach space

Aissa Nasli Bakir
Hassiba Benbouali University of Chlef

In this talk, we present some recent results on an extension of the familiar Theorem of Fuglede-Putnam which asserts that each bounded linear operator which intertwines two normal operators defined on a separable complex Hilbert space, then it intertwines their adjoints too. The extension is studied on (p,k)-quasihyponormal with w-hyponormal operators, log-hyponormal and class Y operators. The given results are then used to study the orthogonality in sense

of Birkhoff, of the range to the kernel of the generalized derivation induced by such operators.

Reconstructing the topology on monoids and clones of the rationals

Mike Behrisch
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Coauthors: John K. Truss, Edith Vargas-García

We study the structures $(\mathbb{Q}, <)$ and $(\mathbb{Q}, \leq)$ through their endomorphism monoids and polymorphism clones. Our main result is that $\mathrm{End}(\mathbb{Q}, <)$ and $\mathrm{End}(\mathbb{Q}, \leq)$ have *automatic homeomorphicity*. That is to say, any monoid isomorphism between the respective endomorphism monoid and any closed transformation monoid on a countable set automatically is a homeomorphism with respect to the natural topology induced by the product topology if the underlying sets are equipped with the discrete topology. Moreover, we reveal a structural property of the endomorphism monoid that allows to extend automatic homeomorphicity to the full polymorphism clone. This method works for $\mathrm{Pol}(\mathbb{Q}, \leq)$, but fails for $\mathrm{Pol}(\mathbb{Q}, <)$.

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Globals of graphs

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Coauthor: Rozalia Madarasz

For every relational structure $\mathcal{A}$ on a set A there is a standard way to construct an appropriate relational structure $\mathcal{P}(A)$ on $\mathcal{P}(A)$. In particular, if G is a graph, the corresponding graph $\mathcal{P}(G)$ will be called the global of the graph G. A class of graphs is globally determined if non-isomorphic graphs from that class have non-isomorphic globals. In this talk we show that some classes of finite graphs, which are important from the point of view of universal algebra, are globally determined.

The second centralizer of a monounary algebra

Miroslava Černegová
P. J. Šafárik University, Košice
Coauthor: Danica Jakubíková-Studenovská

The results concern monounary algebras and their first and second centralizer. We will list necessary and sufficient conditions under which the equality of the second centralizers $C_2(A, f) = C_2(A, g)$ implies the equality of algebras $(A, f) = (A, g)$. In particular, the problem is discussed whether and to what extent the original algebra is determined by its second centralizer. We will find conditions for an operation f when the operation is determined uniquely. In this case, there is an algorithm for a description of f.

Lattices with unique complementation

Ivan Chajda
Palacký University Olomouc

It is well known that every complemented distributive lattice is uniquely complemented. The converse does not hold in general. It is only known that every atomic uniquely complemented lattice is distributive (Birkhoff-Ward Theorem, Ogasawara-Sasaki Theorem). It was shown by V.N.Salij in 1972 that there exists an infinite uniquely complemented lattice which is not distributive. We presented a necessary and sufficient condition formulated in two identities in two variables under which every lattice with complementation becomes distributive uniquely complemented lattice. We show that the distributive law can be easily syntactically derived from these identities.

On the lattice structure of Foulis semigroups

Jānis Cīrulis
University of Latvia, Riga

A Foulis semigroup, known also as a Baer *-semigroup, is an involutive semigroup S with zero in which the right annihilator of any element is generated by a (necessarily unique) projection, i.e., self-adjoint idempotent. These projections are said to be closed; it is known that all closed projections form an orthomodular lattice L. Given an order on S, a subset of S is said to be compatible if every pair of its elements has an upper bound in S. Main theorem: In every Foulis semigroup S, the natural order of closed projections can be extended to the whole S so that any maximal compatible subset $M \subseteq S$ becomes

a lattice isomorphic to an ideal of L, which coincides with L whenever M has the greatest element. Consequently, every initial segment of S is isomorphic to an initial segment of L.

The varieties of Rickart rings and reduced Rickart rings

Insa Cremer
University of Latvia, Riga

A ring R is called a right Rickart ring if for each $a \in R$ there is some idempotent $e \in R$ such that for all $x \in R$, $ax = 0$ if and only if $ex = x$. A reduced ring is a ring without non-zero nilpotent elements. It is known that the class of commutative Rickart rings is a variety. We show that the same holds for the classes of Rickart rings and reduced Rickart rings.

Almost all strongly connected semicomplete digraphs have only trivial idempotent polymorphisms

Petar Đapić
University of Novi Sad

We outline the proof that all strongly connected semicomplete digraphs, except for the 2-cycle (= undirected edge) and the 3-cycle, have only trivial idempotent polymorphisms. The proof uses a complicated induction with several base cases and five types of inductive steps. The result is the key step in our joint work with Petar Marković and Barnaby Martin, where we proved a trichotomy theorem for the complexity of the Quantified Constraint satisfaction Problem over semicomplete templates.

Application of Lattice-theoretical Methods to the existence of equilibrium in ordinal games

Messaoud Deghdak
Université Frères Mentouri, Constantine

The Berge's strong equilibrium is one of Nash's equilibrium refinements which is stable with respect to the deviation of all players except one. Earlier contributions on the existence of Berge's strong equilibrium were based on the Kakutani or Browder-Fan fixed point theorems of correspondences. This requires a topological structure on the strategy space of players, convexity and compactness of the strategy subset of each player and a quasi- concavity and continuity of payoffs functions. In this work, we study the existence of Berge's strong equilibrium and its order structure in non- cooperative game. We introduce the notion of quasi-supermodular ordinal games (the strategy sub-set of each player is a lattice and its preferences are represented by a quasi-modular weak order on his strategy sub-set) to the context of Berge's strong equilibrium. We prove the existence of Berge's strong equilibrium which turns out to be a complete lattice. The application of Zhou's fixed point theorem instead of classical fixed point avoids the use of standard assumptions on the game.

Finer complexity of Constraint Satisfaction Problems with Maltsev Templates

Dejan Delic
Ryerson University, Toronto
Coauthor: Aklilu Habte

In this talk, we will sketch a proof that there is a first-order reduction of solvability of constraint satisfaction problems with finite Maltsev templates to the problem of solving systems of equations over finite simple affine modules. This yields a proof of the fact that constraint satisfaction problems over finite Maltsev templates are in complexity classes $\mathrm{MOD}_k\mathrm{L}$ for a suitable $k \geq 2$, which depends on the template.

Quasigroups with few associative triples

Aleš Drápal
Charles University in Prague

Let Q be a quasigroup of order n. A triple (a, b, c) of its elements is said to be associative, if $a(bc) = (ab)c$. No asymptotic construction seems to have been described for which the number of associative triples were less than n^2. Reasons for such a regretable state of affairs will be described, and some recent progress will be reported.

Invariance groups of lattice-valued functions

Eszter K. Horváth
University of Szeged
Coauthors: Branimir Šešelja, Andreja Tepavčević

In the first part of the talk some general observations will be presented about the so called *cuts* of lattice-valued functions (that are mappings from S to L where S is a set and L is a lattice). Then we give some very short information about the invariance group problem, i.e. about the group of those permutations of variables, that leave an n-variable function invariant. We show that the invariance group of a lattice-valued n-variable function on the k-element set depends only on the cuts of the function, i.e. on the canonical representation of the function. Furthermore, we construct such lattice-valued Boolean function (and its generalization), the cuts of which represent all representable invariance groups.

Complemented quasiorder lattice of a monounary algebra

Lucia Janičková
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If A is an algebra, then the set consisting of all reflexive and transitive relations on A, which are compatible with all operations of A (i.e., quasiorders of A), will be denoted Quord A. Then Quord A is a lattice with respect to inclusion. It is easy to see that the latice Con A of all congruences of A is a sublattices of Quord A. We will deal with the lattice Quord(A, f) of all quasiorders of (A, f), where (A, f) is a monounary algebra. Monounary algebras (A, f) whose lattices of quasiorders are complemented have been characterized as follows: (C) $f(x)$ is a cyclic element for all $x \in A$, and all cycles have the same square-free

number n of elements. The condition (C) is necessary and sufficient. Analogical conditions for the lattice $\mathrm{Con}(A, f)$ were proved by Egorova and Skornyakov. There is introduced the notation of a connected quasiorder. Let (A, f) be a monounary algebra satisfying (C). We will present an algorithm constructing a complement to any connected $q \in \mathrm{Quord}(A, f)$. Next a construction of a complement is performed for any $q \in \mathrm{Quord}(A, f)$.

On rings which are sums of two PI-subrings

Marek Kępczyk
Bialystok University of Technology

Let R be an associative ring and let R_1, R_2 be subrings of R such that $R = R_1 + R_2$, i.e. for every $r \in R$ there exist $r_1 \in R_1$ and $r_2 \in R_2$ such that $r = r_1 + r_2$. In 1995 K. I. Beidar and A. V. Mikhalev stated the following problem: suppose that R_1 and R_2 satisfy polynomial identities (shortly, are PI-rings), is then also R a PI ring? In this talk, some new results concerning the Beidar-Mikhalev problem will be presented.

CSPs over the random partial order

Michael Kompatscher
Vienna University of Technology
Coauthor: Trung Van Pham

We study computational problems of the form Poset-SAT(Φ): In such problems the input consists of finitely many variables and constraints about them taken from Φ, a fixed set of quantifier-free formulas in the language of orders. The question is, whether there is a partial order satisfying all the constraints. We show that, for every Φ, Poset-SAT(Φ) is either in P or NP-complete. To prove this result we first restate the Poset-SAT problems as constraint satisfaction problems (CSPs) of reducts of the random partial order. We then apply the universal algebraic approach, i.e. we study the polymorphism clones of the reducts instead of the reducts themselves. To do so we use Ramsey methods developed by Bodirsky and Pinsker. It turns out that whenever such a polymorphism clone contains a weak-nu term modulo endomotphisms, the corresponding CSP is in P. All other CSPs on reducts of the random partial order are NP-complete.

The radicals of local residuated lattices

Michiro Kondo
Tokyo Denki University

We consider properties of local additive measures on local residuated lattices in terms of filters and prove the characterization of these residuated lattices. We also provide a characterization theorem of the notion of relative free of zero divisors. It follows from our result that every local residuated lattice X relative free of zero divisors can be simply represented by $X = Rad(X) \cup \{0\}$ and hence that every local additive measure on X can be extend to unique 2-valued Riečan state and it is the only one Riečan state. At last, we prove that every local residuated lattice is strong.

Divisibility and torsion in commutative semirings with several generators

Miroslav Korbelář
Czech Technical University, Prague

A commutative ring is called almost divisible iff every element is divisible with respect to some infinite set of prime numbers. Finitely generated commutative rings that are almost divisible have to be torsion. It seems that this property may be extended to finitely generated commutative semiring as well. We will discuss the case connected with small number of generators.

On rings whose set of nilpotent elements is a Wedderburn radical subring

Kamil Kozłowski
Bialystok University of Technology
Coauthor: Ryszard Mazurek

For an associative ring R (not necessarily with identity), let $N(R)$ denote the set of nilpotent elements of R. The set $N(R)$ can provide important information on the structure of the ring R. For example, if R is reduced (i.e., if $N(R) = \{0\}$), then R is a subdirect product of domains. In this talk we will discuss properties of rings R for which the set $N(R)$ is a subring. We will pay special attention to the class of rings R such that $N(R)$ is a Wedderburn radical subring of R. This class includes many important types of rings (e.g., Armendariz rings, semicommutative rings).

On the size of concept lattices after generalizing attributes

Leonard Kwuida
Bern University of Applied Sciences
Coauthors: Temgoua Etienne, Kuiche Rostand

To control the number of concepts of a binary context, many approaches have been proposed. One of this is to replace some attributes by their union. We call this new attribute a generalized attribute. This operation reduces the size of the attribute set, but not automatically the size of the concept lattice. We will discuss the size of concept lattices of such contexts in our talk.

Relatively pseudocomplemented posets

Helmut Länger
Vienna University of Technology
Coauthor: Ivan Chajda

The notion of a relative pseudocomplement is generalized from meet-semilattices to posets. Surprisingly, many properties remain valid. Relatively pseudocomplemented posets satisfying a certain identity turn out to be join-semilattices. Relations between distributivity of posets and the existence of relative pseudocomplements are presented.

Characterization of Modularity by Means of Cover-Preserving Sublattices

Marcin Łazarz
University of Wrocław

J. Jakubík in the paper "Modular Lattice of Locally Finite Length" (Acta Sci. Math., 37, 1975) proved that a lattice of locally finite length is modular if and only if it contains no cover-preserving sublattice isomorphic to S_7 nor to S_7^* nor to $N_{m,n}$ for $m \geq 4$, $n \geq 3$ (so called a centered hexagon, a dual centered hexagon, a cell, respectively). A straightforward consequence of this theorem is an earlier result of F. Šik which states, that an upper semimodular lattice of locally finite length is modular if and only if it contains no cover-preserving sublattice isomorphic to S_7. In the talk I present a sketch of the proof of the Šik's theorem extended to the class of upper continuous and strongly atomic lattices. Moreover, I construct a counterexample which shows that it is impossible to give an analogous extension of Jakubík's theorem.

Permutation classes closed under pattern involvement and composition

Erkko Lehtonen
Dresden University of Technology

For an arbitrary subgroup G of the symmetric group S_ℓ, we determine the permutations involving only members of G as ℓ-patterns. For any $n \geq \ell$, the set of all n-permutations with this property constitutes again a permutation group. In this way, we refine and strengthen the classification of sets of permutations closed under pattern involvement and composition that is due to Atkinson and Beals.

Monoid of Nd-Full Hypersubstituions

Somsak Lekkoksung
Rajamangala University of Technology, Isan Khon Kaen Campus

An nd-full hypersubstitution maps any operation symbols to the set of full terms of type τ_n. Nd-full hypersubstitutions can be extended to mappings which map sets of full terms to sets of full terms. The aim of this talk is to show that the extension of an nd-full hypersubstitution is an endomorphism of some clone and show that the set of all nd-full hypersubstitutions forms a monoid.

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Characterizations of Idempotent Elements in Pre-Generalized Hypersubstitutions of type $\tau = (m, n)$

Nareupanat Lekkoksung
Khon Kaen University
Coauthor: Prakit Jampachon

A generalized hypersubstitution of type $\tau = (m, n)$ is a mapping σ which maps the m-ary operation symbol f and n-ary operation symbol g to the term $\sigma(f)$ and $\sigma(g)$, and may not preserved arities. Each generalized hypersubstitution can be extended to a mapping $\hat{\sigma}$ on the set of all terms of type $\tau = (m, n)$. The structure $(\mathrm{Hyp}_{\mathrm{G}}(\tau); \circ_{\mathrm{G}}, \sigma_{\mathrm{id}})$ is a monoid where σ_{id} is an identity hypersubstitution. A pre-generalized hypersubstitution of type $\tau = (m, n)$, namely σ, is a generalized hypersubstitution of type $\tau = (m, n)$ where $\sigma(f)$ and $\sigma(g)$ are not variables. In this talk, we present some conditions that a pre-generalized hypersubstitution of type $\tau = (m, n)$ is idempotent.

On the complexity of Quantified Constraint Satisfaction Problem via polymorphisms

Petar Marković
University of Novi Sad

This lecture will state and briefly explain the significance of the Quantified Constraint Satisfaction Problem (QCSP) with fixed template, a kind of decision problems which are always within the class Pspace. The analogue of the famous Dichotomy Conjecture for CSP in the case of QCSP is that the only possible complexities are P, NP-complete and Pspace-complete (depending on the template). We give some criteria for Pspace-completeness and for being within the class NP. These, along with some ideas specific to the case under investigation, enabled Djapić, Martin and the speaker to prove that semicomplete digraphs exhibit trichotomy. We will mention some newer results. The lecture will end with an open problem (unless we solve it by the time of the lecture).

Finiteness conditions for subdirect products

Peter Mayr
University of Colorado, Boulder
Coauthor: Nik Ruškuc

We consider when the property of being finitely generated and finitely presented is preserved under subdirect products for various algebraic structures (in particular, Mal'cev algebras, idempotent algebras, and algebras in congruence modular varieties).

An idempotent groupoid having no Taylor term and no cube-term blockers

Ralph McKenzie
Vanderbilt University, Nashville

Matthew Moore (unpublished) has shown that any variety of idempotent algebras has a cube-term if and only if its 2-freely-generated algebra admits no "coloring" to the structure $\langle\{0,1\}, R_n(n \geq 1)\rangle$ where $R_n = \{0,1\}^n \setminus \{(1,\ldots,1)\}$. The result encourages a suspicion that the coloring condition can be replaced by the condition that a generating algebra $\mathbf{A}$ for the variety has a "cube-term blocker", or weaker, that the 2-freely generated algebra in the variety has a cube-term blocker (as is known to be true for a variety $V(\mathbf{A})$ where $\mathbf{A}$ is finite).

In fact, if $\mathbf{A}$ is infinite, it may both fail to have a cube term, and fail to possess a cube-term blocker. There are very easy to construct examples of such $\mathbf{A}$ of infinite signature. The example I will talk about is an infinite idempotent groupoid $\mathbf{W}$. No equation other than equations that hold in every idempotent groupoid, holds in any two-generated subalgebra of $\mathbf{W}$. So $\mathbf{W}$, and any one of its two-generated subalgebras $\mathbf{A}$, has no Taylor term, a fortiori no cube term.

There are no cube-term blockers in these algebras. In fact, for any two distinct elements $a, b \in W$ there are terms $s(x,y), t(x,y)$ which induce on $\{a,b\}$ the operations of a two-element lattice: $s(a,b) = s(b,a) = a$, $t(a,b) = t(b,a) = b$.

Duality for dyadic intervals

Anna Mućka
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Coauthors: Anna Romanowska, Katarzyna Matczak

Dyadic rationals are rationals whose denominator is a power of 2. Dyadic n-dimensional polytopes are defined as the intersections with the n-dimensional dyadic space of an n-dimensional real polytope whose vertices lie in the dyadic space. The one-dimensional analogs are dyadic intervals. Algebraically, they carry the structure of a commutative, entropic and idempotent algebra under the binary operation of arithmetic mean. There are infinitely many (pairwise non-isomorphic) dyadic intervals. Our main result describes a duality for the class of dyadic intervals. The duality is given by an infinite schizophrenic object, the dyadic unit interval. The dual spaces are certain subgroupoids of the square of the dyadic unit interval with additional constant operations.

On algebras with a linear bound on the length of terms

Nebojša Mudrinski
University of Novi Sad
Coauthor: Erhard Aichinger

We define the length of a term as the number of operational symbols and variables that occur in the term. In general, for a given finite algebra, it is not easy to determine the minimal length of n-ary terms that suffices to represent all different n-ary term functions of the algebra. We have conjectured that supernilpotency decides whether the bound is polynomial or exponential. We have started with certain abelian (1-supernilpotent) algebras. If an abelian algebra has a prime number of elements p and one fundamental operation from the clone of $(\mathbb{Z}_p, +)$ we obtain that the bound is linear.

Varieties with Property Going Up

Claudia Mureşan
University of Bucharest
Coauthor: George Georgescu

This is a sequel of the set of results we have announced at the 91st Edition of the Workshop on General Algebra. Let $\mathcal{C}$ be a congruence–modular equational class, so that the commutator, $[\cdot,\cdot]$, is defined in $\mathcal{C}$ (see [1], [5]). For any member A of $\mathcal{C}$, we denote by $\mathrm{Spec}(A)$ the set of the *prime congruences* of A, that is those congruences ϕ of A such that, for any congruences α, β of A, $[\alpha,\beta] \subseteq \phi$ implies $\alpha \subseteq \phi$ or $\beta \subseteq \phi$. Let A and B be members of $\mathcal{C}$, $f : A \to B$ be a morphism in $\mathcal{C}$, and f^* be the inverse image of the morphism $f \times f : A \times A \to B \times B$. We say that f is *admissible* iff $f^*(\mathrm{Spec}(B)) \subseteq \mathrm{Spec}(A)$. In the case when f is admissible, we say that f fulfills the *Going Up* property (abbreviated *GU*) iff, for all $\psi \in \mathrm{Spec}(B)$, $\{\phi \in \mathrm{Spec}(A) \mid f^*(\psi) \subseteq \phi\} \subseteq f^*(\{\chi \in \mathrm{Spec}(B) \mid \psi \subseteq \chi\})$, and we say that f fulfills the *Lying Over* property (abbreviated *LO*) iff $\{\phi \in \mathrm{Spec}(A) \mid \mathrm{Ker}(f) \subseteq \phi\} \subseteq f^*(\mathrm{Spec}(B))$. Out of the results from the first part of this research, we recall that: surjectivity implies admissibility, but the converse is not true, admissibility doesn't always hold, admissibility does not imply LO, and GU implies LO. Now we shall focus on equational classes with all morphism being admissible, and fulfilling GU (thus also LO). Note that, according to [7], each variety with equationally definable principal congruences (EDPC) is congruence–distributive. We have proved that, in any variety with EDPC, all admissible morphisms fulfill GU, thus also LO. In any discriminator variety, all morphisms are admissible, hence all morphisms fulfill GU and LO, since each discrimin ator variety has EDPC, according to [6, Theorem 3.2, p. 389]. Discriminator varieties include Boolean algebras, Post algebras, n–valued MV–algebras, monadic algebras, cylindric algebras, Gödel residuated lattices [8] etc.. See many examples of varieties with EDPC in [3]. A result from [4] shows that any congruence–distributive variety with the principal intersection property (PIP) has EDPC. We have shown that, in any congruence–distributive variety with PIP, all morphisms are admissible, hence all morphisms fulfill GU and LO, by the above. It is well known that lattices and residuated lattices are congruence–distributive. By [2, Example 4.7, p. 120], the variety of distributive lattices has PIP; it is immediate that so does the variety of residuated lattices. To collect the main results above, we conclude that examples of equational classes with all morphisms admissible and fulfilling GU and LO include discriminator va rieties and congruence–distributive varieties with PIP, the latter of which include distributive lattices and residuated lattices.

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Taylor's modularity conjecture for idempotent identities

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One of the open problems stated in Taylor's and Garcia's monograph on the lattice of interpretability types of varieties is to answer the following conjecture which is usually refered to as the Taylor's modularity conjecture: The union of every two sets of identities in disjoint signatures implies the existence of Gumm terms only if one of the two original sets does. We will show that this is true for sets of identites which imply that every operation is idempotent.

On products of amalgams and amalgams of products

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Coauthor: Christian Pech

We study amalgamated free sums in strict amalgamation classes of finite relational structures that are closed with respect to finite products. Examples of such classes are the class of finite partial orders, the class of non-empty finite metric spaces, the class of finite simple graphs,...

In particular, we are interested in the interaction between amalgamated free sums and direct products. It turns out that there is a canonical homomorphism

$$h : (\mathbf{B} \times \mathbf{B}') \oplus_{\mathbf{A}\times\mathbf{A}'} (\mathbf{C} \times \mathbf{C}') \to (\mathbf{B} \oplus_{\mathbf{A}} \mathbf{C}) \times (\mathbf{B}' \oplus_{\mathbf{A}'} \mathbf{C}').$$

When is this canonical homomorphism an embedding?
In this talk we will present results toward the general answer to this question.

Complexity classification for the semilinear-order constraint satisfaction problems

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The universal homogeneous binary tree $(S_2; \leq)$ is defined as the Fraissé limit of finite semilinear orders. A reduct of $(S_2; \leq)$ is a structure that is first-order definable in $(S_2; \leq)$. Recently, the automorphism groups of the reducts of $(S_2; \leq)$ are classified by Bodirsky, Bradley-Willians, Pinsker and Pongrácz. By using this result we give a full complexity classification of the constraint satisfaction problems for the reducts of $(S_2; \leq)$. The classification uses the results and techniques from various research areas: universal algebra, Leeb's Ramsey theorem for rooted trees and the classification of the automorphism groups of the reducts of $(S_2; \leq)$.

Algebras with a central semilattice operation

Agata Pilitowska
Warsaw University of Technology

We present algebras of the form $(A, \Omega, +)$, where (A, Ω) is an algebra of a given type and $+$ is a join-semilattice operation which commutes with all basic operations of (A, Ω). Examples of such algebras are given by semilattice modes (idempotent and entropic algebras), which play an essential rôle in the classification of finite modes and were investigated by K. Kearnes. We show that (similarly as for semilattice modes) to each variety of an idempotent algebras with a central semilattice operation one can associate a semiring whose structure determines some properties of the variety.

The algebraic dichotomy conjecture for infinite domain Constraint Satisfaction Problems

Michael Pinsker
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We prove that an ω-categorical core structure primitively positively interprets all finite structures with parameters if and only if some stabilizer of its polymorphism clone has a homomorphism to the clone of projections, and that this happens if and only if its polymorphism clone does not contain operations α, β, s satisfying the identity $\alpha s(x,y,x,z,y,z) \approx \beta s(y,x,z,x,z,y)$. This establishes an algebraic criterion equivalent to the conjectured borderline between P and NP-complete CSPs over reducts of finitely bounded homogenous structures, and accomplishes one of the steps of a proposed strategy for reducing the infinite domain CSP dichotomy conjecture to the finite case. Our theorem is also of independent mathematical interest, characterizing a topological property of any ω-categorical core structure (the existence of a continuous homomorphism of a stabilizer of its polymorphism clone to the projections) in purely algebraic terms (the failure of an identity as above).

Diagram induced topological properties of congruence lattices

Miroslav Ploščica
P. J. Šafárik University, Košice

We define special separation properties in Priestley spaces, induced by commutative diagrams of finite distributive semilattices. We connect the liftability of such diagram by the Con functor in a variety $\mathcal{V}$ with the failure of the corresponding separation property in the congruence space of some algebra in $\mathcal{V}$. We use these properties for proving that the critical point between two finitely generated congruence-distributive varieties with Compact Intersection Property can be at most $\aleph_1$. As a tool, we find a new theorem of infinite combinatorics concerning free sets.

Permutation groups, permutation patterns, and Galois connections

Reinhard Pöschel
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Coauthor: Erkko Lehtonen

Consider a permutation $\pi \in S_n$ as a string $\pi_1 \dots \pi_n$ where $\pi_i = \pi(i)$ for all $i \in \{1, \dots, n\}$ and hence $\{\pi_1, \dots, \pi_n\} = \{1, \dots, n\}$. A permutation $\sigma \in S_\ell$ with $\ell \leq n$ that is obtained from a substring of π of length ℓ by compressing it to numbers $1, \dots, \ell$ while keeping unchanged the relative order of the elements is called an (ℓ-)*pattern* of π. In this talk we mainly consider a monotone Galois connection (gComp-gPat) arising from the notion of permutation patterns. The corresponding Galois closures (kernels, resp.) characterize those permutation groups in S_n (S_ℓ, resp.) whose patterns belong to a given subgroup of S_ℓ (which are generated by the patterns of a given subgroup of S_n, resp.). These closures and kernels can be described as automorphism groups of special ℓ-ary relations.

A variety of orthocomplemented lattices related to Z_2-valued measures

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Let L be an orthocomplemented lattice (OCL) and let $M(L)$ be the set of all Z_2-group-valued measures. Then L is called Z_2-rich if for any $x, y \in L$, $x \not\leq y$ there exists an $m \in M(L)$ such that $m(u) = 1$ for each $u \in [x, 1_L]$ and $m(v) = 0$ for each $v \in [0_L, y]$. It is first seen that any Z_2-rich OCL is orthomodular. Then it is found out that the class of Z_2-rich OCLs is a (rather interesting) variety. The talk discusses several properties of this variety.

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Derivations on GMV-algebras

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GMV-algebras are a non-commutative generalization of MV-algebras and are an algebraic semantics of the non-commutative Łukasiewicz infinite valued propositional fuzzy logic. We investigate derivations on GMV-algebras (which are formally introduced in the same manner as derivations on rings). We give a complete description of all derivations on any GMV-algebra.

The Cycle Structure of Quandles

NAQEEB UR REHMAN
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A quandle is a selfdistributive algebraic structure with three axioms which are related to three Reidemeister moves of knot diagrams. The binary operation of quandle structure is like the conjugation in a group. The quandle structure can be studied as sequences of permutations. The cycle structure of permutations of indecomposable quandles is well-behaved because permutations of an indecomposable quandle are mutually conjugate and hence have same cycle structures. In [1], C. Hayashi observed another interesting property of the cycle structure of an indecomposable quandle and conjectured that the permutation of an indecomposable quandle has cycles whose cycle lengths divide the largest among them. In this talk, the classes of indecomposable quandles for which Hayashi's conjecture is true will be discussed. An obstruction on the cycle structure of certain indecomposable quandles will also be provided in this talk.

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Lattices without absorption

Anna Romanowska
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A bisemilattice is an algebra with two (possibly different) structures of a semilattice (idempotent commutative semigroup) defined on the same set. The class of bisemilattices contains lattices (defined by the absorption laws: $x + xy = x(x + y) = x$), and stammered semilattices (defined by the law: $x + y = xy$), where both semilattice operations coincide. In this talk we will be interested in bisemilattices close to lattices, called Birkhoff systems (defined by the "almost absorption" law: $x + xy = x(x + y)$). I will discuss some methods of constructing Birkhoff systems from lattices and semilattices.

Topological index of some Carbon Nanotubes and Symmetry Group for Nanotubes and unit cells in solid

Hamid Saati Shirvan
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The Padmakar–Ivan (PI) index of a graph G is defined as $PI(G) = \sum[\mathrm{neu}(e \mid G) + \mathrm{nev}(e \mid G)]$, where $\mathrm{neu}(e \mid G)$ is the number of edges of G lying closer to u than to v, $\mathrm{nev}(e \mid G)$ is the number of edges of G lying closer to v than to u and summation goes over all edges of G. Let G be a connected graph, $\mathrm{Meu}(e \mid G)$ is the number of vertices of G lying closer to u and $\mathrm{Mev}(e \mid G)$ is the number of vertices of G lying closer to v. Then the Szeged index of G is defined as the sum of $\mathrm{Meu}(e \mid G)\,\mathrm{Mev}(e \mid G)$, over edges of G. The PI index of G is the Szeged-like topological index defined as the sum of $[\mathrm{neu}(e \mid G) + \mathrm{nev}(e \mid G)]$, where $\mathrm{neu}(e \mid G)$ is the number of edges of G lying closer to u than to v, $\mathrm{nev}(e \mid G)$ is the number of edges of G lying closer to v than to u and summation goes over all edges of G. Types of symmetry groups are commonly used in chemistry. Point groups are used for molecules, whereas, for solids, the 230 space groups are used. Neither of these types of symmetry groups are suitable for representing unit cells in solids. The symmetry which is intermediate between that of point groups and space group represent the symmetry of unit cells in an infinite lattice, a third type of symmetry group must be used. An algorithmic method of generating these symmetry groups is described. It can be demonstrated that these groups are valid by use of conventional symmetry group theory, this technique has been applied to the two-dimensional graphite lattice. Because the new method generates symmetry tables using only the topology of the system, the symmetry

properties of graphs can also readily be derived. Last, the relationship between these groups and the other two types of groups is identified.

Keimel's Problem and threshold convexity

Jonathan Smith
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Barycentric algebras are semilattice sums of convex sets that are fundamental to the analysis of complex and biological systems. For example, they have been applied to the fuzzy logic of cells in systems biology, and to multilevel competition models that combine demography with ecology. Barycentric algebras have a binary operation for each element of the open real unit interval, subject to the (hyper)identities of idempotence, skew-commutativity, and skew-associativity. Since skew-associativity is rather complicated, Klaus Keimel has asked whether it can simply be replaced by the entropic (hyper)identity in the axiom set for barycentric algebras. In this talk (joint work with A.B. Romanowska), we present counterexamples showing that the answer to Keimel's Problem is negative. The counterexamples are interesting in their own right, using a new concept of threshold convexity to create an entire spectrum of algebras, stretching from barycentric algebras at one end to commutative, entropic magmas at the other.

On Epimorphisms of Ordered Algebras

Nasir Sohail
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Flatness properties and amalgamation of monoids are known to undergo several restrictions if a compatible order is introduced on top of the algebraic structure. For instance the three-element chain semilattice, that is absolutely flat and hence an amalgamation base in the class of all monoids, fails to retain these properties in the ordered context for its nine out of thirteen compatible orders. We, however, proved in 2015 that epimorphisms of monoids (and semigroups) are not affected by the introduction of order. In this talk I shall present my recent work which shows that in fact epimorphisms of any algebra are not affected by the introduction of order. We shall also need to consider dominions and special amalgamation.

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On representation of lattice-valued frames as quantale algebras

Sergejs Solovjovs
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Some time ago, I. Stubbe has constructed an isomorphism between the categories of right Q-modules and cocomplete skeletal Q-categories for a given unital quantale Q. Employing similar technique, we obtain an isomorphism between the categories of Q-algebras (which is a crisp notion, motivated by the concept of algebra over a commutative ring with identity) and Q-quantales (which is a lattice-valued analogue of the concept of quantale), where Q is additionally assumed to be commutative. As a consequence, we provide a common framework for two concepts of lattice-valued frame, which are currently available in the literature. Moreover, we obtain a convenient setting for lattice-valued extensions of the famous equivalence between the categories of sober topological spaces and spatial locales, as well as for answering the question on the relationships of this equivalence to the notion of stratification of lattice-valued topological spaces.

Group coextensions of monoids in an ordered setting

Thomas Vetterlein
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A coextension of a monoid S is a monoid E together with a surjective homomorphism $\pi\colon E \to S$. If the kernel of π is contained in $\mathcal{H}$, the coextension belongs to the so-called group coextensions of monoids, which were defined by P. A. Grillet and J. Leech in the 1970's and can be described in a transparent way. We are interested in coextensions of pomonoids, that is, monoids that are additionally endowed with a compatible partial order. The coextensions discussed in this context in the literature are apparently often based on similar ideas; we have in particular in mind the theory of triangular norms, which are operations used in fuzzy logic and give rise to pomonoids on the real unit interval. We propose a theory similar to Grillet's and Leech's, but having regard to the

compatible order. We moreover generalise the construction, allowing monoids instead of groups to be employed as extending structures.

Code loops in dimension up to 8

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Code loops are combinatorial objects built from intersection properties of vectors in doubly even binary codes that play an important role in the construction of local subgroups of sporadic groups. I will report on a computational project in which we have extended enumeration of code loops from dimension 5 to dimension 8 (order 512).

On the poset of minors of a function

Tamás Waldhauser
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Let $\mathcal{F}_{AB} = \{f\colon A^n \to B \mid n \in \mathbb{N}\}$ denote the set of all B-valued functions of several variables defined on A, and let us introduce a quasiorder $\preceq$ and the corresponding equivalence relation $\equiv$ on $\mathcal{F}_{AB}$ by

$$g \preceq f \iff g(x_1,\dots,x_k) = f(x_{i_1},\dots,x_{i_n}) \text{ for some } i_1,\dots,i_n \in \{1,\dots,k\};$$
$$g \equiv f \iff g \preceq f \text{ and } f \preceq g.$$

If $g \preceq f$, then g is said to be a *minor* of f, and it is easy to see that this holds if and only if g can be obtained from f by identifying variables, permuting variables and adding or deleting inessential variables. Our main goal is to describe the structure of the poset of all minors of a given function, i.e., the principal ideals in $(\mathcal{F}_{AB}/{\equiv};\preceq)$. We give a characterization of these "minor posets" in terms of colorings of partition lattices, and we use these colorings to give examples and constructions of minor posets. Despite these results, the most basic question remains open: Is there a finite bounded poset that cannot occur as the poset of all minors of a function?

On quasivarieties of graphs

Anna Zamojska-Dzienio
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A quasivariety **K** is a class of algebraic systems closed under isomorphisms, subsystems, direct products, and ultraproducts. The quasivarietes contained in a quasivariety **K** form a complete lattice **Lq(K)** under inclusion. In this talk we describe the structure of a quasivariety lattice of graphs which is highly complex. In 1997 A. V. Kravchenko showed that a quasivariety of graphs **G** is *Q-universal*, i.e. for every quasivariety **M** the lattice **Lq(M)** is a homomorphic image of some sublattice of **Lq(G)**. We have shown that **Lq(G)** is *unreasonable* which means there is no algorithm to decide whether a given finite lattice embeds into such a quasivariety lattice. There are also uncountably many quasivarieties of graphs which have undecidable quasi-equational theory and for which the finite membership problem is undecidable.

Automorphisms of the category of free commutative dimonoids

Yurii Zhuchok
Taras Shevchenko National University of Kyiv

An algebra $(D, \dashv, \vdash)$ with two binary associative operations $\dashv$ and $\vdash$ is called a *dimonoid* [1] if for all $x, y, z \in D$ the following conditions hold:

$$(x\dashv y)\dashv z = x\dashv(y\vdash z),\ \ (x\vdash y)\dashv z = x\vdash(y\dashv z),\ \ (x\dashv y)\vdash z = x\vdash(y\vdash z).$$

A dimonoid $(D, \dashv, \vdash)$ is called *commutative* [2] if $x \dashv y = y \dashv x$, $x \vdash y = y \vdash x$ for all $x, y \in D$. A more general information on dimonoids, the construction of a free commutative dimonoid and examples of other dimonoids can be found, e.g., in [1], [2]. We denote by $\mathfrak{FC}_X$ the free commutative dimonoid generated by X and use the denotation $\mathfrak{FC}_1$ for the case $|X| = 1$. By $\mathbb{N}$ we denote the set of all natural numbers and let $\mathbb{N}_{\tilde{2}} = \mathbb{N} \cup \{\tilde{2}\}$.

Proposition 1. *The endomorphism monoid $End(\mathfrak{FC}_1)$ of the free commutative dimonoid $\mathfrak{FC}_1$ is isomorphic to the semigroup $(\mathbb{N}_{\tilde{2}}, \odot)$, where $\odot$ is defined by*

$$\tilde{2} \odot \tilde{2} = 4,\ \ m \odot n = mn,\ \ m \odot \tilde{2} = \tilde{2} \odot m = \begin{cases} \tilde{2}, & m = 1, \\ 2m, & m \neq 1. \end{cases} \quad (m, n \in \mathbb{N})$$

Proposition 2. *Let X be a singleton set, Y be an arbitrary set and $End(\mathfrak{F}_X) \cong End(\mathfrak{F}_Y)$. Then $|Y| = 1$ and isomorphisms of $End(\mathfrak{F}_X)$ onto*

$End(\mathfrak{F}_Y)$ are in a natural one-to-one correspondence with permutations $f : P_{\tilde{2}} \to P_{\tilde{2}}$ of the set P of all prime numbers with $\tilde{2}$ such that

$$f|_{\{2,\tilde{2}\}} = \begin{pmatrix} 2 & \tilde{2} \\ 2 & \tilde{2} \end{pmatrix} \text{ or } f|_{\{2,\tilde{2}\}} = \begin{pmatrix} 2 & \tilde{2} \\ \tilde{2} & 2 \end{pmatrix}.$$

By $S(X)$ we denote the symmetric group on a set X, and by C_2 the 2-element group.

Proposition 3. *All automorphisms of the monoid $End(\mathfrak{FC}_1)$ are inner. In addition, the automorphism group of $End(\mathfrak{FC}_1)$ is isomorphic to the direct product $S(P) \times C_2$.*

Theorem 1. *If endomorphism semigroups $End(\mathfrak{FC}_X)$ and $End(\mathfrak{FC}_Y)$ are isomorphic, then free commutative dimonoids $\mathfrak{FC}_X$ and $\mathfrak{FC}_Y$ are isomorphic too.*

An automorphism $\Phi : End(\mathfrak{FC}_X) \to End(\mathfrak{FC}_X)$ is called *quasi-inner* if there exists a permutation α of $\mathfrak{FC}_X$ such that $\beta\Phi = \alpha^{-1}\beta\alpha$ for all $\beta \in End(\mathfrak{FC}_X)$. If α turns out to be an automorphism of $\mathfrak{FC}_X$, *Phi* is an inner automorphism of $End(\mathfrak{FC}_X)$.

Theorem 2. *For an arbitrary set X with $|X| \geq 2$, all automorphisms of $End(\mathfrak{FC}_X)$ are quasi-inner. In addition, the automorphism group $Aut(End(\mathfrak{FC}_X))$ is isomorphic to $S(X) \times C_2$.*

Note that the automorphism group of the endomorphism monoid of a free semigroup or a free monoid was described by Mashevitsky G. and Schein B.M. [3]. The description of automorphisms of the category of free abelian dimonoids was announced in [4].

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On CSP Dichotomy Conjecture

Dmitriy Zhuk
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At the previous AAA conference I presented an algorithm that solves a constraint satisfaction problem on 5-element domain in polynomial time if all constraints are preserved by a weak near-unanimity operation. This time I present a modified version of the algorithm, and formulate a condition that is sufficient for the algorithm to work in general. This condition holds for small domains and possibly holds for any domain.

On extensions of multiary maps to superposition of binary ones

Pasha Zusmanovich
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In 1944 Stefan Banach asked: whether a commutative ternary multiplication can be extended to superposition of a binary multiplication? We will discuss several interpretations of this question in the context of Polish mathematics of that time, touching upon such diverse subjects as multivalued logics, binary and ternary semigroups, theory of clones, and Hilbert's 13th problem. We will present proofs of a few simple results along these lines, basing on an old idea of Jacobson about envelopes of Lie triple systems, and ask related combinatorial questions.