Directed Jónsson and Gumm terms

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Directed Jónsson and Gumm terms are variants of the classic terms for congruence distributive and congruence modular varieties. Given an $n \in \mathbb{N}$, the chain of length n of directed Jónsson terms is:

$$\begin{aligned} d_1(x,x,y) &\approx x, & \\ d_i(x,y,x) &\approx x & \text{for } 1 \le i \le n, \\ d_i(x,y,y) &\approx d_{i+1}(x,x,y) & \text{for } 1 \le i < n, \\ d_n(x,y,y) &\approx y, & \end{aligned}$$

while the chain of $n+1$ directed Gumm terms is:

$$\begin{aligned} d_1(x,x,y) &\approx x, & \\ d_i(x,y,x) &\approx x & \text{for } 1 \le i \le n, \\ d_i(x,y,y) &\approx d_{i+1}(x,x,y) & \text{for } 1 \le i < n, \\ d_n(x,y,y) &\approx p(x,y,y) & \\ p(x,x,y) &\approx y. & \end{aligned}$$

Besides being aestheticaly pleasant, these directed terms are often more comfortable to use than the classic ones; directed Gumm terms appear eg. in Libor Barto's proof of the Valeriote conjecture.

It is straightforward to show that if a variety V admits directed Jónsson resp. Gumm terms then V is congruence distributive resp. modular. We will explain how to go about proving the converse implication. This is a joint work with Marcin Kozik, Ralph McKenzie, and Matt Moore.